# MATH-423 Rings and Fields

**Credit Hours:** 3

**Prerequisite:** MATH- 325 Group Theory-I

**Course Objectives:** In abstract algebra, ring theory is the study of rings, an algebraic structure in which addition and multiplication are defined and have similar properties to those operations defined for the integers. Ring theory studies the structure of rings and their representations. Ring theory was originated in mid-nineteenth century by Richard Dedekind.

**Core Contents:** Rings, subrings, integral domains, fields, ideals, factor rings, polynomial rings, ring homomorphisms, field extensions, finite fields.

Detailed Contents: Rings, properties of rings, ring of Gaussian integers, subrings, subring test, zero devisors, integral domains, fields, finite integral domains, characteristic of a ring, ideals, ideal test, principal ideal, prime ideals, maximal ideals, factor ring, existence of factor ring, ring homomorphism, ring isomorphism, properties of ring homomorphism, kernel of a homomorphism, natural homomorphism from ring to its factor ring, the field of quotients, polynomial rings, reducibility and irreducibility tests, principal ideal domains, construction of finite fields, extension fields, the fundamental theorem of field theory, splitting fields, algebraic extensions,

**Course Outcomes:** On successful completion of this course, students will know rings, properties of rings, ring of Gaussian integers, subrings, subring test, zero devisors, integral domains, fields, finite integral domains, characteristic of a ring, ideals, principal ideal, prime ideals, maximal ideals, factor ring, ring homomorphism, ring isomorphism, properties of ring homomorphism, kernel of a homomorphism, applications of first isomorphism theorem, the field of quotients, polynomial rings, reducibility and irreducibility tests, principal ideal domains, finite fields, extension fields, the fundamental theorem of field theory, splitting fields. algebraic extensions, degree of an extension, finite extensions.

**Textbook:** J. A. Gallian, Contemporary Abstract Algebra. 8th ed. Brooks/Cole, Belmont, CA, 2013.

**Reference Books:**


1. W. Keith Nicholson, Introduction to Abstract Algebra, (3rd edition), 2007, John Wiley & sons.
2. J. B. Fraleigh, A first course in abstract algebra (7th edition), 1998, Addison-Wesley publishing.
3. I.N. Herstein, Topics in Algebra (2nd edition), New York, John Wiley & sons, Inc., 1975.


| **Weekly Breakdown** |
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| Week | Section | Topics |
|---|---|---|
| 1 | Ch.12 | Rings, definition and examples, properties of rings, uniqueness of identity and inverses, direct sum of rings. |
| 2 | Ch.12 | Subrings, definition and examples, subring test, Gaussian integers, applications of subring tests. The group of unit elements of a commutative ring. Boolean ring. |
| 3 | Ch.13 | Zero divisors, integral domains, cancellation property with respect to multiplication, fields, finite integral domains. The ring of integers modulo a prime *p*. |
| 4 | Ch.13 | Characteristic of a ring, characteristic of a ring with unity, characteristic of an integral domain, Ideals, ideal test, |
| 5 | Ch.14 | Examples related to ideal test, principal ideals, Factor ring, existence of factor ring. |
| 6 | Ch.14 | Prime ideals, maximal ideals, examples, and related theorems, idempotent elements, nilpotent elements, and related results. |
| 7 | Ch.15 | Ring homomorphism, examples, ring isomorphism, Properties of ring homomorphism, kernels and ideals, the first isomorphism theorem for rings. |
| 8 | Ch.15 | Ideals and kernels, homomorphism from the ring of integers to a ring with unity and its consequences. The field of quotients. |
| 9 | **Mid Semester Exam** | |
| 10 | Ch.16 | The ring of polynomials over a commutative ring, the ring of polynomials over integral domains, the division algorithm for ring of polynomials over a field, factor theorem, remainder theorem. |
| 11 | Ch.16 & Ch. 17 | Principal ideal domain, polynomial ring over a field is a principal ideal domain, reducible and irreducible polynomials over an integral domain. |
| 12 | Ch.17 | Reducibility tests, reducibility tests for degree 2 and 3, content of a polynomial, primitive polynomial, Gauss's Lemma, reducibility over ring of rational numbers implies reducibility over ring of integers. |
| 13 | Ch.17 | Irreducibility tests, mod p irreducibility tests, examples, Eisenstein's criterion, irreducibility of *p*-th cyclotomic polynomial, the rational root theorem. |
| 14 | Ch.17 | Characterization of maximal ideals in a ring of polynomials over a field, construction of fields. |
| 15 | Ch.19 & Ch.20 | Revision of vector space, Extension fields, fundamental theorem of field theory, splitting fields |
| 16 | Ch.21 | Algebraic extensions. |
| 17 | | Review |
| 18 | **End Semester Exam** | |